DEHRADUN PUBLIC SCHOOL
ASSIGNMENT 2019-20
SUBJECT- MATHEMATICS (041)
CLASS -XII

RELATION AND FUNCTIONS

1. If \( f(x) = (5-x^2)^{1/2} \), then find \( fof(x) \).
2. If a function \( f: A \to [-6, \infty) \) given by \( f(x) = 9x^2 + 6x - 5 \) is invertible, find \( f^{-1}(x) \).
3. Show that the relation \( S \) in the Set \( A = \{5,6,7,8,9\} \) given by \( S=\{(a,b): |a-b| \text{ is divisible by } 2 \} \) is an equivalence relation. Find the set of all elements related to 6.
4. Let \( A = \{1, 2, 3, 4\} \) and \( R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (3, 2)\} \). Show that \( R \) is reflexive and transitive but not symmetric.
5. Let \( f(x) = x + 7 \) and \( g(x) = x - 7, x \in \mathbb{R} \). Find \( (fog)(7) \).
6. Let \( R_+ \) be the set of all positive real numbers. Let \( f: R_+ \to [4, \infty] : f(x) = x^2 + 4 \). Show that \( f \) is invertible and find \( f^{-1} \). (CBSE 2013)
7. Prove that the function \( f: \mathbb{N} \to \mathbb{N} \) defined by \( f(x) = x^2 + x + 1 \) is one-one but not onto.
8. If \( f(x) = \sqrt{x^2 + 1} \) and \( h(x) = 2x-3 \), then find \( f^{-1} \left( h^{-1}(g^{-1}(x)) \right) \). (CBSE 2015)
9. Consider \( f: R_+ \to [-9,\infty) \) given by \( f(x)=5x^2+6x-9 \). Prove that \( f \) is invertible with \( f^{-1}(y) = \left( \frac{\sqrt{54+5y-3}}{5} \right) \). (CBSE 2015)
10. If the function \( f: \mathbb{R} \to \mathbb{R} \) be defined by \( f(x) = x^2 + 5x + 9 \) Find \( f^{-1}(9) \).
11. Show that \( f: \mathbb{R} \to \mathbb{R} \) , defined by \( f(x) = \sin x \), is neither one – one nor onto.
12. Show that the function \( f: \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = \frac{x}{x^2+1} \forall x \in \mathbb{R} \), is neither one – one nor onto.
13. If \( f: \mathbb{R} \to (0,2) \) defined by \( f(x) = \sin^{-1} + 1 \) is invertible, find \( f^{-1} \).
14. If function \( f \) and \( g \) are given by \( f = \{(1, 2), (3, 5), (4, 1), (2,6)\} \), \( g= \{(2, 6), (5,4), (1, 3), (6, 1)\} \), find the range of \( f \) and \( g \) and write down the function \( fog \) and \( gof \).

INVERSE TRIGONOMETRIC FUNCTIONS

1. Show that \( \tan^{-1} \left( \frac{\sin^{-1} \frac{3}{4}}{2} \right) = \frac{4-\sqrt{7}}{3} \). (EXAMPLER)
2. Solve \( \cos (\tan^{-1} x) = \sin (\cot^{-1} \frac{3}{4}) \). (CBSE 2013)
3. Prove that \( \tan^{-1} \frac{63}{16} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5} \).
4. Prove that \( \cos [\tan^{-1} (\sin(Cot^{-1} x))] = \sqrt{1+x^2} \). (CBSE 2010, 2008)
5. Prove that \( \tan^{-1} \left( \frac{1}{3} \right) + \tan^{-1} \left( \frac{1}{5} \right) + \tan^{-1} \left( \frac{1}{7} \right) + \tan^{-1} \left( \frac{1}{8} \right) = \frac{\pi}{4} \). (CBSE 2011,2014)
6. Prove that \( \cot^{-1} \left[ \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right] = \frac{x}{2} , x \in (0,\pi) \). (CBSE 2011)
7. Prove that \( \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left( \frac{1}{3} \right) = \frac{9}{4} \sin^{-1} \left( \frac{2\sqrt{2}}{3} \right) \). (CBSE 2011)
8. Prove that \( \tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2} = \tan^{-1} \left( \frac{3x-x^2}{1-3x^2} \right), x^2 < \frac{1}{3} \). (CBSE 2010)
9. Prove that \( \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi \). (DELHI CBSE 2010)
10. If \( y = \cot^{-1} (\sqrt{\cos x}) \) - \( \tan^{-1} (\sqrt{\cos x}) \) prove that \( \sin y = \tan^2 \frac{x}{2} \). (FOREIGN 2013)
11. Solve for \( x \) : \( \tan^{-1}(x+1)+\tan^{-1}(x-1) = \tan^{-1} \frac{34}{31} \). (CBSE 2015)
12. Prove the following : \( \cot^{-1} \left( \frac{xy+1}{x-y} \right) + \cot^{-1} \left( \frac{yz+1}{y-z} \right) + \cot^{-1} \left( \frac{zx+1}{z-x} \right) = 0 \). (CBSE 2015)
13. Solve for x: \( \sin^{-1}6x + \sin^{-1}6\sqrt{3}x = -\frac{\pi}{2} \)  
(CBSE Sample Paper 2015)

14. Solve the equation for x: \( \sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x \)  
(CBSE 2016)

15. If \( \cos^{-1}x_a + \cos^{-1}x_b = \alpha \), prove that \( \frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha \)  
(CBSE 2016)

16. Find the greatest and least value of \( (\sin^{-1}x)^2 + (\cos^{-1}x)^2 \)  
(Exampler)

17. Prove that \( \cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right) = 7 \)

18. If \( \tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5} \), then find the value of \( \cot^{-1}x + \cot^{-1}y \)

19. Solve: \( \cos^{-1}(\sin (\cos^{-1}x)) = \frac{\pi}{3} \)

**MATRICES AND DETERMINANTS**

1. Use elementary column operation \( C_2 \rightarrow C_2 + 2C_1 \) in the following matrix equation:
\[
\begin{pmatrix}
2 & 1 \\
2 & 0
\end{pmatrix}
= \begin{pmatrix}
3 & 1 \\
2 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
1 & 1
\end{pmatrix}
\]  
(CBSE 2016)

2. Construct a 2x2 matrix whose element \( a_{ij} \) are given by \( \frac{(2i + j)^2}{i^2} \).

3. If A is a square matrix of order 3 such that \( |adj A| = 289 \) find \( |A| \)

4. If \( A^2 = A \) find value of (\( 1 + A \))^2 - 3A

5. If \( A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \) then prove that \( A^n = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}, n \in \mathbb{N} \)

6. Using \( E_{R_j} \) transformation find the inverse of \( A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix} \)  
(CBSE 2009)

7. If \( A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix} \), find \( A^{-1} \) and hence solve the system of linear equations:
\[
x + 2y - 3z = -4; 
2x + 3y + 2z = 2; 
3x - 3y - 4z = 11.
\]

8. If \( A = \begin{pmatrix} -2 & -1 & -2 \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \), find \( A^{-1} \). Using \( A^{-1} \), solve the system of linear equations: \( x - 2y = 10, \)
\[
2x - y - z = 8, 
-2y + z = 7
\]

9. Use the product \( \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix} \) to solve the following system of equations:
\[
x - y + 2z = 1; 
2y - 3z = 1; 
3x - 2y + 4z = 2.
\]
(CBSE 2011)

10. Find the adjoint of the matrix \( A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \) and hence show that \( A.(adj A) = | A | I_3 \)
(CBSE 2015)

11. Prove that \[
\begin{vmatrix}
a^2 + 1 & ab & ac \\
ab & b^2 + 1 & bc \\
ca & cb & c^2 + 1
\end{vmatrix}
= (1 + a^2 + b^2 + c^2)
\]
(CBSE 2014, 2009, 2013)

12. Evaluate using properties of determinant
\[
\begin{vmatrix}
1 + a^2 - b^2 & 2ab & -2b \\
2ab & 1 - a^2 + b^2 & 2a \\
2b & -2a & 1 - a^2 - b^2
\end{vmatrix}
= (1 + a^2 + b^2)^3
\]
(CBSE 2013, CBSE 2008, 2009)

13. Using properties of determinants, prove the following
\[
\begin{vmatrix}
a^2 & bc & ac + c^2 \\
ab & b^2 + bc & c^2 \\
ab & b^2 & c^2
\end{vmatrix}
= 4a^2b^2c^2
\]
(CBSE 2015)
14. Using the properties of determinants, show that $\Delta ABC$ is isosceles if:
\[
\begin{vmatrix}
1 & 1 & 1 \\
1 + \cos A & 1 + \cos B & 1 + \cos C \\
\cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C
\end{vmatrix} = 0
\] (CBSE 2016)

15. If $x, y, z$ are in GP then using properties of determinants where $x \neq y \neq z$ and $P$ is any real number,
\[
\begin{vmatrix}
px + y & x & y \\
y & py + z & y \\
0 & px + y & py + z
\end{vmatrix} = 0
\] (CBSE Sample Paper 2015)

16. Without expanding, show that
\[
\begin{vmatrix}
cosec^2 \theta & \cot^2 \theta & 1 \\
\cot^2 \theta & cosec^2 \theta & -1 \\
42 & 40 & 2
\end{vmatrix} = 0
\] (Exampler)

17. Show that if the determinant of
\[
\begin{vmatrix}
3 & -2 & \sin 3\theta \\
-7 & 8 & \cos 3\theta \\
11 & 14 & 2
\end{vmatrix} = 0 , \sin \theta = 0 \text{ or } \frac{1}{2}
\] (Exampler)

18. Find the number of distinct real roots of
\[
\begin{vmatrix}
\cos x & \sin x & \cos x \\
\cos x & \cos x & \sin x \\
\cos x & \sin x & \cos x
\end{vmatrix} = 0 \text{ on the interval } -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}
\]

19. If $x = -4$ is a root of
\[
\begin{vmatrix}
x & 2 & 3 \\
1 & x & 1 \\
3 & 2 & x
\end{vmatrix} = 0 , \text{ then find the two other roots.}
\] (Exampler)

20. To promote the making of toilets for women, an organization tried to generate awareness through
(i) house calls (ii) letters, and (iii) announcements. The cost for each mode per attempt is given as
: (i) Rs. 50 (ii) Rs. 20 (iii) Rs. 40

The numbers of attempts made in three villages X, Y, and Z are given below:

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>400</td>
<td>300</td>
<td>100</td>
</tr>
<tr>
<td>Y</td>
<td>300</td>
<td>250</td>
<td>75</td>
</tr>
<tr>
<td>Z</td>
<td>500</td>
<td>400</td>
<td>150</td>
</tr>
</tbody>
</table>

Find the total cost incurred by the organisation for the three villages separately, using matrices. Write one value generated by the organisation in the society. (CBSE 2015)

21. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards. (CBSE 2013)

22. Using properties of determinants, prove that
\[
\begin{vmatrix}
1 & 1 & 1 + 3x \\
1 + 3y & 1 & 1 \\
1 & 1 + 3z & 1
\end{vmatrix} = 9 (3xyz + xy + yz + zx)
\] (CBSE 2017)

23. Using elementary row transformations, find the inverse of the matrix
\[
A = \begin{bmatrix}
1 & 2 & 3 \\
2 & 5 & 7 \\
-2 & -4 & -5
\end{bmatrix}
\] (CBSE 2017)
CONTINUITY & DIFFERENTIABILITY

1. Find the constants a and b so that the function
   \[ f(x) = \begin{cases} 
   3ax + b, & x > 1 \\
   11, & x = 1 \\
   5ax - 2b, & x < 1
   \end{cases} \]
   is continuous at \( x = 1 \). (DELHI CBSE 2011)

2. Find the value of k if the function
   \[ f(x) = \begin{cases} 
   \frac{1 - \sin x}{(\pi - 2x)^2} & x \neq \pi/2 \\
   k & x = \pi/2
   \end{cases} \]
   is continuous at \( x = \pi/2 \).

3. If \( y = \frac{1 - \sin 2x}{1 + \sin 2x} \), show that \( \frac{dy}{dx} + \sec^2(x - \pi) = 0 \).

4. If the function \( f(x) \) defined below is continuous at \( x = 0 \). Find the value of k.
   \[ f(x) = \begin{cases} 
   \frac{1 - \cos 2x}{2x^2} & x < 0 \\
   k & x = 0 \\
   \frac{x}{|x|} & x > 0
   \end{cases} \]
   (CBSE 2010)

5. Find the value of k so that the function \( f(x) = \begin{cases} 
   \frac{2^{x+2} - 16}{4^x - 16} & x \neq 2 \\
   k & x = 2
   \end{cases} \)
   is continuous at \( x = 2 \).

6. Find values of \( p \) and \( q \), for which \( f(x) = \begin{cases} 
   \frac{1 - \sin^3 x}{3 \cos^2 x} & x < \frac{\pi}{2} \\
   p & x = \frac{\pi}{2} \\
   \frac{q(1 - \sin x)}{(x - 2)^2} & x > \frac{\pi}{2}
   \end{cases} \)
   is continuous at \( x = \frac{\pi}{2} \).

7. If \( f(x) = \begin{cases} 
   \frac{\sin(a+1)x + 2 \sin x}{x} & x < 0 \\
   \frac{2}{\sqrt{1 + bx - 1}} & x = 0
   \end{cases} \)
   is continuous at \( x = 0 \), the find the values of \( a \) and \( b \).

8. If \( f(x) = \frac{\sqrt{2} \cos x - 1}{\cot x - 1} \), \( x \neq \frac{\pi}{4} \) find the value of \( f(\frac{\pi}{4}) \) so that \( f(x) \) becomes continuous at \( x = \frac{\pi}{4} \).

9. If \( y = x \sin(a + y) \), prove that \( \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a} \). (DELHI CBSE 2012, CBSE 2009, 2013)

10. If \( x^y = e^{x-y} \), prove that \( \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2} \). (CBSE 2013)

11. If \( x = \sqrt{a^x - 1}, y = \sqrt{a \cos^{-1} t} \), show that \( \frac{dy}{dx} = \frac{-y}{x} \). (CBSE 2012)

12. If \( y = 3e^{2x} + 2e^{3x} \), prove that \( \frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0 \). (CBSE 2009)

13. If \( y = \log [x + \sqrt{x^2 + a^2}] \). Prove that \( (x^2 + a^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0 \). (DELHI CBSE 2013)

14. Differentiate w.r.t. \( x \) : \( \sin^{-1} \left( \frac{2x^3 + 3x}{1 + (36)^2} \right) \).
15. Show that the function $f(x) = |x-1| + |x+1|$, for all $x \in \mathbb{R}$, is not differentiable at the points $x = -1$ and $x = 1$. (CBSE 2015)

16. Find $\frac{dy}{dx}$ if $x = \frac{1+\log t}{t^2}$, $y = \frac{3+2 \log t}{t}$. (Exampler)

17. Find $\frac{dy}{dx} : \cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$. (Exampler)

18. If $\cos x \cos x \cos x$, show that $\frac{dy}{dx} = \frac{y^2 \tan x}{y \log \cos x - 1}$. (Exampler)

19. If $x = a (2 \theta - \sin 2 \theta )$ and $y = (1 - \cos 2 \theta )$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$. (CBSE 2017)

APPLICATION OF DERIVATIVES

1. A spherical soap bubble is expanding so that its radius in increasing at the rate of 0.02 cm/sec. At what rate is the surface area increasing when its radius is 5 cm. ($\pi = 3.14$)

2. A rocket is ascending vertically at the rate of 1000km/hr. If the radius of the earth is $R$ km, how fast is the area (A) of the earth, visible from the rocket, increasing 15 minutes after it started ascending? Given that $A = \frac{2\pi R^2 H}{R + H}$, where $H$ is the height of the rocket above the surface of the earth.

3. Prove that the curve $x = y^2$ and $xy = k$ cut at right angles of $8 k^2 = 1$. (CBSE 2008)

4. Find the equation of the tangent and the normal to the curve $x = 1 - \cos \theta$, $y = \theta - \sin \theta$ at $\theta = \frac{\pi}{4}$. (CBSE 2010)

5. Show that the equation of normal at any point on the curve $x = 3\cos \theta - \cos^3 \theta$, $y = 3\sin \theta - \sin^3 \theta$ is $4 (y \cos^3 \theta - x \sin^3 \theta) = 3 \sin 4 \theta$. (Exampler)

6. Find the maximum and minimum values of $f(x) = \sec x + \log \cos^2 x$, $0 < x < 2\pi$. (Exampler)

7. Prove that $y = \log(1 + x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of $x$ throughout its domain. (CBSE 2012)

8. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is strictly increasing or strictly decreasing.

9. Separate the interval $[0, \frac{\pi}{2}]$ into sub intervals in which $f(x) = \sin^4 x + \cos^4 x$ is (a) Increasing (b) decreasing.

10. Find interval in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or decreasing. (2016)

11. Find C of the mean value theorem for the function $f(x) = 2x^2 - 10x + 29$ is $[2, 7]$.

12. Verify the Rolle’s theorem if $f(x) = \sin^4 x + \cos^4 x$ in $[0, \frac{\pi}{2}]$.

13. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius $R$ is $\frac{2R}{\sqrt{3}}$. Find the volume of the largest cylinder inscribed in a sphere of radius $R$.

14. Prove that the perimeter of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

15. The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue (in rupees) received from the sale of $x$ units of a product is given by $R(x) = 3x^2 + 36x + 5$, Find the marginal revenue, when $x = 5$, and write value does the question indicate. (CBSE 2013)

16. An open box with a square base is to be made out of a given quantity of card board of area $c^2$ square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units. (CBSE 2005)
16. Find the area of greatest rectangle that can be inscribed in an ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) (Exampler)

17. An isosceles triangle of vertical angle 2\( \theta \) is inscribed in a circle of radius \( a \). Show that the area of triangle is maximum when \( \theta = \frac{\pi}{6} \) (Exampler)

18. If the sum of the surface areas of cube and a sphere is constant, what is the ratio of an edge of the cube to the diameter of the sphere, when the sum of their volumes is minimum. (Exampler)

19. AB is a diameter of a circle and C is any point on the circle. Show that the area of triangle ABC is maximum, when it is isosceles. (Exampler)

20. The sum of the surface areas of a rectangular parallelepiped with sides \( x \), \( 2x \)and \( \frac{x}{3} \) and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if \( x \) is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes. (Exampler)

21. Find the equation of the tangent and the normal, to the curve \( 16x^2 + 9y^2 = 145 \) at the point \((x_1, y_1)\), where \( x_1 = 2 \) and \( y_1 > 0 \) (CBSE 2017)

22. Find the intervals in which the function \( f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12 \) is (a) strictly increasing (b) strictly decreasing. (CBSE 2017)

### INTEGRALS

Evaluate:

<table>
<thead>
<tr>
<th>No.</th>
<th>Integral</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \int \frac{5x+3}{\sqrt{x^2+4x+10}} , dx )</td>
<td>( \int \frac{(x^2+1)e^x}{(x+1)^2} , dx )</td>
</tr>
<tr>
<td>3</td>
<td>( \int \frac{1+\tan x}{(x+\log sec x)} , dx )</td>
<td>( \int \cos x \cos 2x \cos 3x , dx )</td>
</tr>
<tr>
<td>5</td>
<td>( \int \sqrt{1+2\tan x} , (\tan x + \sec x) , dx )</td>
<td>( \int \frac{\sin x}{(2+\cos x)(5+\cos x)} , dx )</td>
</tr>
<tr>
<td>7</td>
<td>( \int \frac{\sin x - \cos x}{\sin 2x} , dx )</td>
<td>( \int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x} , dx ) (Exampler)</td>
</tr>
<tr>
<td>9</td>
<td>( \int \tan^2 x sec^4 x , dx )</td>
<td>( \int \sqrt{\cot x + \sqrt{\tan x}} , dx ) (CBSE 2014)</td>
</tr>
<tr>
<td>11</td>
<td>( \int dx )</td>
<td>( \int (3 - 2x) \cdot \sqrt{2 + x - x^2} ) (CBSE 2015)</td>
</tr>
<tr>
<td>13</td>
<td>( \int \frac{x}{x^4 + x^2 - 2} , dx )</td>
<td>( \int \frac{x^3}{x^4 + 3x^2 + 2} ) (Exampler)</td>
</tr>
<tr>
<td>15</td>
<td>( \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} , dx )</td>
<td>( \int \frac{\sin x + \cos x}{(\sin x + \cos x)^2} , dx )</td>
</tr>
<tr>
<td>17</td>
<td>( \int \frac{2\sin x}{3\sin x + 4\cos x} , dx )</td>
<td>( \int \frac{2\sin x + 3\cos x}{5\sin x + 4\cos x} , dx )</td>
</tr>
<tr>
<td>19</td>
<td>( \int \frac{1}{4 \cos x - 3 \sin x} , dx )</td>
<td>( \int \frac{1}{5 + 3 \sin^2 x} , dx )</td>
</tr>
<tr>
<td>21</td>
<td>( \int_1^3 (2x^2 + 3) , dx ) as limit of Sum.</td>
<td>( \int_0^3</td>
</tr>
<tr>
<td>23</td>
<td>( \int_{\pi/2}^\pi \frac{\sin^2 x}{\sin x + \cos x} , dx )</td>
<td>( \int_0^{\pi/4} \log(1 + \tan \theta) , d\theta )</td>
</tr>
<tr>
<td>27</td>
<td>( \int_0^a \sin^{-1} \left( \frac{x}{a+x} \right) , dx )</td>
<td>( \int_0^{\pi/2} \sin 2x \tan^{-1} (\sin x) , dx )</td>
</tr>
<tr>
<td>29</td>
<td>( \int_{-2}^2 x \cos x , dx )</td>
<td>( \int_{-2}^2 \frac{\sqrt{10-x}}{\sqrt{x+10} - x} , dx ) (Exampler)</td>
</tr>
<tr>
<td>31</td>
<td>( \int_{-\pi/2}^{\pi/2} \sin x + \cos x , dx )</td>
<td>( \int_{-\pi/2}^{\pi/2} \frac{2\cos x}{(1 - \sin x)(1 + \sin^2 x)} , dx ) (CBSE 2017)</td>
</tr>
</tbody>
</table>
APPLICATIONS OF INTEGRALS

1. Using Integration, compute the area bounded by the lines, \( x + 2y = 2 \), \( y - x = 1 \) and \( 2x + y = 7 \).
2. Using integration find the area of triangular region whose vertices are \((1,0),(2,2),(3,1)\).
3. Find the area of the region bounded by \( y^2 = 4x \), \( x = 1 \), \( x = 4 \) and \( x\)-axis in the first quadrant.
4. Find the area bounded by the curve \( x^2 = 4y \) and the straight line \( x = 4y - 2 \). (DELHI CBSE 2010, 2013)
5. Find the area of the region bounded by the parabola \( y = x^2 \) and \( y = |x| \). (CBSE 2013)
6. Find the area of the region bounded by the parabola \( y^2 = 4ax \) and \( x^2 = 4ay \). (FOREIGN 2013)
7. Using integration find the area of region enclosed between the circles \( x^2 + y^2 = 1 \) and \((x-1)^2 + y^2 = 1\).
8. Sketch the graph of the curve \( y = |x + 3| \). Find evaluate \( \int_{-6}^{0} |x + 3| \, dx \).

9. Using integration, find the area of the triangle formed by positive \( x \) axis and the tangent and normal to the circle \( x^2 + y^2 = 4 \) at \((1,\sqrt{3})\) (2015)
10. Using integration, find the area bounded by the curves \( y = |x - 1| \) and \( y = 3 - |x| \) (2015)
11. Sketch the region bounded by the curves \( y = \sqrt{5 - x^2} \) and \( y = |x - 1| \) and find the area using integration (CBSE sample paper)
12. Using integration, find the area of the region bounded by the line \( x - y + 2 = 0 \), the curve \( x = \sqrt{y} \) and \( y \) axis. (2015)
13. Using integration, find the area of the region in the first quadrant enclosed by the \( x\)-axis, the line \( y = x \) and the circle \( x^2 + y^2 = 32 \) (CBSE 2017)

DIFFERENTIAL EQUATION

Find the order and degree

1. \( \frac{d^3y}{dx^3} + 5 \left( \frac{dy}{dx} \right)^2 + \cos y = \tan x \)

Solve the differential equation

2. \((x^2 + xy)dy = (x^2 + y^2)dx\)
3. \(x \frac{dy}{dx} = y \log y - \log x + 1\)
4. \(x^2 \frac{dy}{dx} + y(x + y)dx = 0 \) given that \( y = 1 \) when \( x = 1 \) (2013)
5. \(\frac{dy}{dx} = \left( \frac{y}{x} + \sin \frac{y}{x} \right)\)
6. \(xdy - ydx = \sqrt{x^2 + y^2} \, dx\) (2005, 2011, Example)
7. \((y + 3x^2) \frac{dx}{dy} = x\) (2011)
8. \(xdy + (y - x^3)dx = 0\) (2011)
9. \(x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x\) (2010, 2014)
10. \(\sqrt{1 + x^2 + y^2} + x^2 y^2 + xy \frac{dx}{dy} = 0\)
11. \(\cos^2 x \frac{dy}{dx} + y = \tan x\) (2008, 2011)
12. \((1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}\) (2002, 2014)
13. \( \frac{dy}{dx} = \cos(x + y) + \sin(x + y) \)  

(Exampler)

14. Find the equation of a curve passing through (2,1) if the slope of the tangent to the curve at any point is \( \frac{x^2 + y^2}{2xy} \)  

(Exampler)

15. Obtain the differential equation of the family of circles passing through the points (a,0) and (-a,0)  

(CBSE sample paper)

16. Form the differential equation of the family of ellipses having foci on y-axis and centre at origin.

17. Show that \( y = ae^{\tan^{-1}x} \) is a solution of the differential equation \( (1 + x^2) \frac{d^2y}{dx^2} + (2x - 1) \frac{dy}{dx} = 0 \)

VECTOR ALGEBRA

1. Find the unit vector in the direction of \( \vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k} \)

2. Find the angle between the vector \( \vec{a} = \hat{i} - \hat{j} + \hat{k} \) and \( \vec{b} = \hat{i} + \hat{j} - \hat{k} \).

3. If \( P(1,5,4) \) and \( Q(4,1,2) \) Find the direction ratios of \( \overrightarrow{PQ} \) and direction cosine.

4. If \( |\vec{a}| = \sqrt{3} \) and \( |\vec{b}| = 2 \) find angle between \( \vec{a} \) and \( \vec{b} \)?

5. If \( \vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \) and \( \vec{b} = 3\hat{i} + \hat{j} - 5\hat{k} \), find a vector parallel to \( \vec{a} + \vec{b} \) and having magnitude 8 unit.

6. Find a vector whose magnitude is 3 unit & which is perpendicular to \( \vec{a} = 3\hat{i} + 3\hat{j} - 4\hat{k} \) and \( \vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k} \).

7. If \( \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k} \) and \( \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k} \), show that \( (\vec{a} + \vec{b}) \parallel (\vec{a} - \vec{b}) \) OR orthogonal to each other.

8. Show that the area of parallelogram having diagonals \( \vec{a} = \hat{i} - 3\hat{j} + 4\hat{k} \) and \( \vec{b} = 3\hat{i} + \hat{j} - 2\hat{k} \) is 5\(\sqrt{3} \) sq. unit

(2008)

9. Show that the vectors \( \vec{a} = 3\hat{i} - 2\hat{j} + \hat{k} \), \( \vec{b} = \hat{i} - 3\hat{j} + 5\hat{k} \) and \( \vec{c} = 2\hat{i} - \hat{j} - 4\hat{k} \) form a right angle triangle.

(2005)

10. Let \( \vec{a} = \hat{i} - \hat{j} \), \( \vec{b} = 3\hat{j} - \hat{k} \) and \( \vec{c} = 7\hat{i} - \hat{k} \). Find a vector \( \overrightarrow{d} \) which is perpendicular to both \( \vec{a} \) and \( \vec{b} \) and \( \vec{c} \cdot \overrightarrow{d} = 1 \).

11. If \( \vec{a} \), \( \vec{b} \) and \( \vec{c} \) are three mutually perpendicular vectors of equal magnitude, prove that the angle which \( (\vec{a} + \vec{b} + \vec{c}) \) makes with any of the vectors \( \vec{a} \), \( \vec{b} \) or \( \vec{c} \) is \( \cos^{-1}(1/\sqrt{3}) \)  


12. If the \( \vec{a} + \vec{b} + \vec{c} = 0 \) and \( |\vec{a}| = 3 \), |\( \vec{b} | = 5 \) and \( |\vec{c}| = 7 \) show that the angle between \( \vec{a} \) and \( \vec{b} \) is 60°  

(2008,2014)

13. If \( \vec{a} \), \( \vec{b} \), \( \vec{c} \) are position vectors of vertices A, B and C of a triangle ABC, show that area of triangle is \( \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| \)

14. If \( \hat{a} \) and \( \hat{b} \) are two unit vectors and Q is the angle between them, then show that \( \sin \theta/2 = 1/2|\hat{a} - \hat{b}| \)

15. If \( \vec{a} = \hat{i} + \hat{j} + 2\hat{k} \) and \( \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k} \), find \( (\vec{a} + 3\vec{b}).(2\vec{a} - \vec{b}) \)  

(2002)

16. The two vectors \( \hat{j} + \hat{k} \) and \( 3\hat{i} - \hat{j} + 4\hat{k} \) represents two sides AB and AC respectively of a triangle ABC.

Find the length of the median through A.

(Exampler)

17. If \( \vec{a} = \hat{i} + \hat{j} + \hat{k} \) and \( \vec{b} = \hat{j} - \hat{k} \), find a vector \( \vec{a}^* \) such that \( \vec{a} \times \vec{c} = \vec{b} \) and \( \vec{a}.\vec{c} = 3 \)  

(Exampler)

18. Find the value of \( \lambda \) for which the vectors \( \vec{a} = 3\hat{i} - 6\hat{j} + \hat{k} \) and \( \vec{a}^* = 2\hat{i} - 4\hat{j} + \lambda \hat{k} \) are parallel.

19. If \( \vec{a}, \vec{b}, \vec{c} \) are three vectors such that \( \vec{a} + \vec{b} + \vec{c} = 0 \) and \( |\vec{a}| = 2 \), |\( \vec{b} | = 3 \), |\( \vec{c} | = 5 \), find value of
20. Show that the points with position vectors \( \vec{a} - 2\vec{b} + 3\vec{c}, 4\vec{a} - 7\vec{b} + 7\vec{c} \) and \(-2\vec{a} + 3\vec{b} - \vec{c}\) are collinear.

**THREE DIMENSIONAL GEOMETRY**

1. Find the angle between the lines \( \frac{x}{1} = \frac{y}{2} = \frac{z}{0} \) and \( \frac{x-1}{3} = \frac{y+5}{2} = \frac{z-3}{1} \).

2. Find the angle between the line \( \frac{x-2}{y+5} = \frac{z-3}{2} \) and the plane \( 3x + 4y + z + 5 = 0 \).

3. Find \( p \) such that \( \frac{x}{1} = \frac{y}{2p} \) and \( \frac{x}{3} = \frac{y}{-3} = \frac{z}{2} \) are perpendicular.

4. Using direction ratios, show that the points \((2,3,4), (-1,-2,1)\) and \((5,8,7)\) are collinear.

5. Find the image of the point \((1,6,3)\) in the line \( \frac{x-1}{2} = \frac{y-2}{3} \).

6. Find the distance of the point \((2,4,-1)\) from the line \( \frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} \). Also find foot of perpendicular and image.

7. The cartesian equation of a line are \( 6x - 2 = 3y + 1 = 2z - 2 \) Find (i) The direction ratios of the line (ii)cartesian equation of a line parallel to this line and passing through the point \((2,-1,-1)\).

8. Find the foot of perpendicular from the point \((0,2,3)\) on the line \( \frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} \) find the length of perpendicular.

9. Find the shortest distance between two lines \( \vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3 - 2t)\hat{k} \) and \( \vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} + (-2s - 1)\hat{k} \).

10. Find the angle between the lines whose direction cosines are given by the equations:

    \[ 3l + m + 5n = 0 \] and \(6mn - 2nl + 5lm = 0\).

11. Find the coordinates of the foot of the perpendicular drawn from a point \(A(1,8,4)\) to the line joining the points \(B(0,-1,3)\) and \(C(2,-3,-1)\).

12. Prove that the lines \(x = py + q, z = ry + s\) and \(x = r'y + s'\) are perpendicular if \( pp + rr + 1 = 0\).

13. Find the equation of the two lines through the origin which intersect the line \( \frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1} \) at angles of \( \frac{\pi}{3} \) each.

14. Find the shortest distance between the lines given by \( \vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k} \) and \( \vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}) \).

15. Find the equation of plane passing through the points \((0,-1,0), (1,1,1)\) and \((3,3,0)\).

16. Find the equation of the plane passing through the point \((-1,-1,2)\) and perpendicular to the planes \(3x + 2y - 3z = 1\) and \(5x - 4y + z = 5\).

17. Find the equation the line passing through the point \(P(-1,3,-2)\) and perpendicular to the line \( \frac{x}{1} = \frac{y}{2} = \frac{z}{3} \) and \( \frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5} \).

18. Find the distance of the point \((1,-2,3)\) from the plane \(x - y + z = 5\) measured parallel to the line.
19. Find the p so that the lines \( \frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \) and \( \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5} \) are at right angles. (2008)

20. Find the equation of the plane through the intersection of planes \( x + 2y + 3z -4 = 0 \) and \( 2x + y - z +5 = 0 \) and perpendicular to the plane \( 5x + 3y + 6z +1 = 0 \). (2007)

21. Find the equation of the plane through the point \((4,-3,2)\) and perpendicular to the line of intersection of the planes \( x-y+2z-3=0 \) and \( 2x-y-3z=0 \). Find the point of intersection of the line \( \vec{r} = \vec{i} + 2\vec{j} - \vec{k} + \lambda(\vec{i} + 3\vec{j} - 9\vec{k}) \) and the plane obtained above. (CBSE sample paper)

22. Prove that \( \vec{a} \cdot ((\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c})) = [\vec{a} \ \vec{b} \ \vec{c}] \) (CBSE sample paper)

23. Find the values of \( a \) so that the following lines are skew: \( \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}, \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{2} \) (CBSE sample paper)

24. Find the equation of the two lines through origin which intersect the line \( \frac{x-3}{2} = \frac{y-3}{1} = \frac{z-4}{5} \) in the plane \( 2x - y + z +3 = 0 \).

25. Find the image of the point having position vector \( \vec{r} \) in the plane \( 2x - 2y + 4z +5=0 \).

26. Find the length and the foot of the perpendicular from the point \((1, \frac{3}{2}, 2)\) to the plane \( 2x-2y+4z+5=0 \).

27. Find the image of the point having position vector \( \vec{i} + 3\vec{j} + 4\vec{k} \) in the plane \( \vec{r} \cdot (2\vec{i} - \vec{j} + \vec{k}) + 3 = 0 \).

28. Find the equation of the plane through the points \((2,1,-1)\) and \((-1,3,4)\) and perpendicular to the plane \( x -2y + 4z = 0 \) (Exampler)

29. Let \( \vec{a} = 4\vec{i} + 5\vec{j} - \vec{k}, \ \vec{b} = \vec{i} - 4\vec{j} + 5\vec{k} \) and \( \vec{c} = 3\vec{i} + \vec{j} - \vec{k} \). Find a vector \( \vec{d} \) which is perpendicular to both \( \vec{c} \) and \( \vec{b} \) and \( \vec{d} \cdot \vec{a} = 21 \) (CBSE 2017)

### LINEAR PROGRAMMING

1. A company uses 3 machines to manufacture and sell two types of shirts- half sleeves and full sleeves. Machines A, B and C take 1 hr, 2 hr, and 1 ¾ hr to make a half sleeve shirt and 2 hrs, 1 hr and 1 3/5 hr to make a full sleeve shirt. The profit on each half sleeve shirt is Re 1:00 and on a full sleeve shirt is Re 1:50. No machine can work for more than 40 hr per week. How many shirts of each type should be made to maximize the company's profit? Solve the problem graphically.

2. The tailors X and Y earn Rs 150 and Rs 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. How many days shall each work. If it is desired to produce at least 60 shirts and 32 pants at a minimum labor costs? Solve the problem graphically. (2005)

3. The Principal of a school wants to buy some colour and books for giving prizes to 15 children. He wants to buy at least 4 of each. A colour box costs Rs 5 where as book costs Rs 10. How many of each should he buy so that expenditure does not exceed Rs 100 and at the same time he can buy max. number of prize?

4. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 unit/kg of vitamin A and 1 unit/kg of vitamin C while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs
50/Kg to produce food I and Rs 20/Kg to produce food II. Find the minimum cost of such a mixture formulate the above LPP mathematically and then solve it. (2011)

5. A grain dealer has Rs 1500 for purchase of rice and wheat A bag of rice and a bag of wheat costs Rs180 and 120 respectively .He has a storage capacity of 10 bags only. He earns a profit of Rs 11 and Rs 8 per bag of rice and wheat respectively. How many bags of each must he buy to make maximum profit?

6. An aeroplane can carry a maximum of 200 passengers a profit of Rs 400 is made on each 1st class ticket and a profit of Rs 300 is made on each 2nd class ticket. The airline reserve at least 20 seats for first class. However at least four times as many passengers prefer to travel by second class then by first class. Determine how many tickets of each type must be sold to maximize profit for the airline. Form an LPP and solve it graphically.

7. In a mid day meal programme, an NGO wants to provide vitamin rich diet to the students of an MCD school. The dietician of the NGO wishes to mix two types of food in such a way that vitamin contents of the mixture contains atleast 8 units of vitamin A and 10 units of vitamin c. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per kg of vitamin A and @ units per kg of vitamin C. It costs Rs 50 per kg to purchase food 1 and Rs 70 per kg to purchase food 2. Formulate the problem as LPP and solve it graphically for the minimum cost of such mixture? (CBSE sample paper)

8. If a young man drives his scooter at a speed of 25 Km/hour, he drives the scooter at a speed of 40 Km/hour, it produces more air pollution and increase his expenditure on petrol to Rs 5 per Km. He has maximum of Rs 100 to spend on petrol and travel a maximum distance in one hour time with less pollution. Express this problem as an LPP and solve it graphically. What value do you find here?

9. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at Rs 100 and Rs 120 per unit respectively, how should he uses his resources to maximize the total revenue? From the above as an LPP and solve graphically. Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate?

**PROBABILITY**

1. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once. (2003)

2. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter a car, a truck is respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver. (2000,2002,2008,2012,2014)

3. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that is a actually a six. (2005,2011,2014)

4. In an examination an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it is 1/6. The probability that he knew the answer to the question given he correctly answered it.

5. A card from a pack of 52 cards is lost from the remaining cards of the pack two cards are drawn and are found to be both spades. Find the probability of the lost card being a spade. (2000,2010)

6. A bag contains 4 yellow, 5 red and another bag contains 6 yellow and 3 red balls. A ball is drawn from the first bag and without see its colour, it is put into the second bag. Find the probability that if now is ball is drawn from the second bag. It is yellow in colour. (2002)
7. A problem in mathematics is given to three students whose chances of solving it are \( \frac{1}{3}, \frac{1}{5}, \frac{1}{6} \) what is the probability that at least one of them solves the problem.

8. A pack of playing cards was found to contain only 51 cards, of the first 13 cards which are examined are all red what is the probability that the missing card is black.

9. A random variable \( x \) has the following probability distribution.

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) )</td>
<td>0</td>
<td>( k )</td>
<td>2( k )</td>
<td>2( k )</td>
<td>3( k )</td>
<td>( k^2 )</td>
<td>2( k^2 )</td>
<td>7( k^2 + k )</td>
</tr>
</tbody>
</table>

Determine (i) \( k \) (ii) \( P(x<3) \) (iii) \( P(x>6) \) (iv) \( P(0<x<3) \).

10. A bag contains 4 green and 6 white balls. Two balls are drawn at random one by one without replacement. If the second ball drawn is white, what is the probability that the first ball is also white? (CBSE sample paper)

11. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of diamond cards drawn. Also, find the mean and the variance of the distribution. (CBSE sample paper)

12. A company has two plants to manufacture motor cycles. 70% motor cycles are manufactured at the first plant, while 30% are manufactured at the second plant. At the first plant, 80% motor cycles are rated of the standard quality while at the second plant, 90% are rated of standard quality. A motor cycle randomly picked up, is found to be of standard quality. Find the probability that it has come out from the second plant. Why riding a motor cycle is riskier than driving other vehicles?

13. Out of a group of 30 honest people, 20 always speak the truth. Two persons are selected at random from the group. Find the probability distribution of the number of selected persons who speak the truth. Also find the mean of the distribution. What values are described in this question?

14. A man known to speak the truth 3 out of 5 times. He throws a die and reports that it is a number greater than 4.
   (i) Find the probability that it is actually a number greater than 4.
   (ii) Write about ‘truth’ as essential human value.

15. A bag contains \((2n + 1)\) coins. It is known that \(n\) of these coins have a head on both sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is \(\frac{31}{42}\), determine the value of \(n\). (Exampler)

16. There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let \(X\) denote the sum of the numbers on two cards drawn. Find the mean and variance of \(X\). (Exampler)

17. Two dice are thrown together. Let \(A\) be the event ‘getting 6 on the first die’ and \(B\) be the event ‘getting 2 on the second die’. Are the events \(A\) and \(B\) independent? (Exampler)

18. Three machines \(E_1, E_2, E_3\) in a certain factory produce 50%, 25% and 25% respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines \(E_1\) and \(E_2\) are defective, and that 5% of those produced on \(E_3\) are defective. If one tube is picked up at random from a day’s production, calculate the probability that it is defective. (Exampler)

19. Four balls are to be drawn without replacement from a box containing 8 red and 4 white balls. If \(X\) denote the number of red balls drawn, find the probability distribution of \(X\). (Exampler)

20. A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. If A starts the game, find the probability of winning the game by A in third throw of the pair of dice. (Exampler)
21. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATA NAGAR.

Q1. Let R be a relation on the set N of natural numbers defined by \( n \rightarrow m \) if \( n \) divides \( m \). Then, R is
(a) Reflexive & symmetric  (b) transitive & symmetric  (c) Equivalence (d) Reflexive, transitive but not symmetric.

Q2. If \( f(x) = x^3 + 3 \), then \( f^{-1}(x) \) is
(a) \( x^{1/3} - 3 \)  (b) \( x^{1/3} + 1 \)  (c) \( (x-3)^{1/3} \)  (d) \( x + 3^{1/3} \)

Q3. If \( g(x) = x^2 + x - 2 \) and \( gof(x) = 4x^2 - 10x + 4 \), then \( f(x) \) is
(a) \( 2x - 3 \)  (b) \( 2x + 3 \)  (c) \( 2x^2 + 3x + 1 \)  (d) \( 2x^2 - 3x - 1 \)

Q4. The value of \( \sin^{-1}(\cos 33\pi/5) \) is
(a) \( 3\pi/5 \)  (b) \( -\pi/10 \)  (c) \( \pi/10 \)  (d) \( 7\pi/5 \)

Q5. If \( 4\cos^{-1}x + \sin^{-1}x = \pi \), then value of \( x \) is
(a) \( 3/2 \)  (b) \( 1/2 \)  (c) \( 3^{1/2}/2 \)  (d) \( 2/3^{1/2} \)

Q6. If a matrix A is both symmetric and skew symmetric, then
(a) A is a diagonal matrix  (b) A is a zero matrix  (c) A is a scalar matrix  (d) A is a square matrix

Q7. If A and B are two matrices such that \( AB = A \) and \( BA = B \), then \( B^2 \) is equal to
(a) B  (b) A  (c) 1  (d) 0

Q8. If A is a matrix of order 3 and \(|A| = 8\), then \(|adjA| =
(a) 1  (b) 2  (c) 8  (d) 64

Q9. If A is a square matrix such that \( A^2 = I \), then \( A^{-1} \) is equal to
(a) A+I  (b) A  (c) 0  (d) 2A

Q10. If \( f(x) = \log\left(\sin x + \cos x\right) \), then the value of \( f\left(\frac{\pi}{4}\right) \) is
(a) 1  (b) 1/2  (c) 0  (d) \( \infty \)

Q11. A cylindrical tank of radius 10 cm is being filled with wheat at the rate of 314 cubic cm/hr. Then the depth of wheat is increasing at the rate of
(a) 1 m/hr  (b) 1.1 m/hr  (c) 0.5 m/hr  (d) 0.1 m/hr

Q12. \( \int_{1+\tan x}^{1} dx =
(a) \log(x+\sin x) + C  (b) \log(\sin x + \cos x) + C  (c) 2\sec^2 x/2 + C  (d) 1/2 [x + \log(\sin x + \cos x)] + C

Q13. \( \int_{0}^{\pi/4} \frac{1}{1+4x^2} dx = \frac{\pi}{8}
(a) \frac{\pi}{2}  (b) 1/2  (c) \frac{\pi}{4}  (d) 1

Q14. The solution of the differential equation \( x \ dx + y \ dy = x^2 \ y \ dy - y^2 \ x \ dx \) is
(a) \( x^2 - 1 = C \ (1+y^2) \)  (b) \( x^2 + 1 = C \ (1-y^2) \)  (c) \( x^3 - 1 = C \ (1+y^2) \)  (d) \( x^3 + 1 = C \ (1-y^3) \)

Q15. If \( |\vec{a} \cdot \vec{b}| = 4, |\vec{a}| = 2 \), then \( |\vec{a} \cdot \vec{b}| = 2 \), then \( |\vec{a}| = 2 \)
(a) \( 6 \)  (b) 2  (c) 20  (d) 8

Q16. The image of the point \( (1, 3, 4) \) in the plane \( 2x - y + 3z = -3 \) is
(a) \( (3, 5, 2) \)  (b) \( (-3, 5, 2) \)  (c) \( (3, 5, -2) \)  (d) \( (3, 5, 2) \)

Q17. The maximum value of \( Z = 4x + 3y \) subjected to the constraints \( 3x + 2y \geq 160, 5x + 2y \geq 200, x + 2y \geq 80; \ x, y \geq 0 \) is
(a) 320  (b) 300  (c) 230  (d) none of these

Q18. The least number of times a fair coin must be tossed so that the probability of getting at least one head is
(a) 7  (b) 6  (c) 5  (d) 3

Key for MCQ
Q1. (d)  Q2. (c)  Q3. (a)  Q4. (b)  Q5. (c)  Q6. (b)  Q7. (a)  Q8. (d)  Q9. (b)  Q10. (a)  Q11. (a)
Q12. (d)  Q13. (b)  Q14. (a)  Q15. (c)  Q16. (b)  Q17. (d)  Q18. (d)